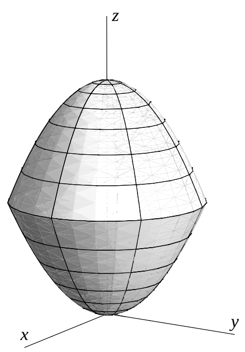
Triple Integrals in Cylindrical or Spherical Coordinates

1. Let $\mathcal{U}$ be the solid enclosed by the paraboloids $z = x^2+y^2$ and $z = 8-(x^2+y^2)$. (Note: The paraboloids intersect where $z = 4$.) Write $\iiint_{\mathcal{U}} xyz\ dV$ as an iterated integral in cylindrical coordinates.

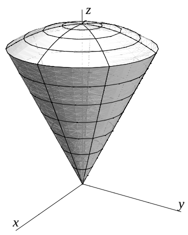


2. Find the volume of the solid ball $x^2 + y^2 + z^2 \leq 1$.

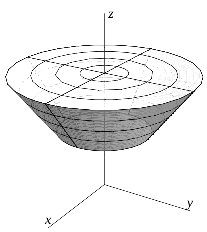
3. Let $\mathcal{U}$ be the solid inside both the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 1$. Write the triple integral $\iiint_{\mathcal{U}} z\ dV$ as an iterated integral in spherical coordinates.

For the remaining problems, use the coordinate system (Cartesian, cylindrical, or spherical) that seems easiest.

4. Let $\mathcal{U}$ be the "ice cream cone" bounded below by $z = \sqrt{3(x^2 + y^2)}$ and above by $x^2 + y^2 + z^2 = 4$. Write an iterated integral which gives the volume of $\mathcal{U}$. (You need not evaluate.)



5. Write an iterated integral which gives the volume of the solid enclosed by $z^2 = x^2 + y^2$, $z = 1$, and $z = 2$. (You need not evaluate.)



6. Let $\mathcal{U}$ be the solid enclosed by $z = x^2 + y^2$ and $z = 9$. Rewrite the triple integral $\iiint_{\mathcal{U}} x \, dV$ as an iterated integral. (You need not evaluate, but can you guess what the answer is?)

7. The iterated integral in spherical coordinates $\int_{\pi/2}^{\pi} \int_0^{\pi/2} \int_1^2 \rho^3 \sin^3 \phi \, d\rho \, d\phi \, d\theta$ computes the mass of a solid. Describe the solid (its shape and its density at any point).